Radiation and Convection in Circular Pipe with Uniform Wall Heat Flux

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The interaction of thermal radiation with laminar forced convection in thermally developing circular pipe flow is analyzed theoretically. The analysis considers an absorbing and emitting gray fluid bounded by a heated black wall having a uniform heat flux subjected at a certain finite length of wall and insulated otherwise. The contribution of thermal radiation is obtained by solving the two-dimensional exact integral equations for the divergence of radiative heat flux with the use of finite element node approximation technique. The governing energy equation is solved numerically by the Crank-Nicolson finite difference method with an iterative procedure. Effects of various radiation and convection parameters such as optical radius, Peclet number, and inlet fluid temperature on wall radiative heat flux are presented. The effects of the parameters on the development of fluid bulk temperature and wall temperature are also analyzed.

Nomenclature

C_p = specific heat
I_b = blackbody intensity
I_w = boundary intensity
k = thermal conductivity
L = dimensionless length of heating section, l/R
l = length of heating section
m = number of radial grid points for convection part
n = number of radial node points for radiation part
$\tilde{n}$ = refractive index
Nc = conduction to radiation parameter, $k\kappa/(4\tilde{n}^2\sigma T_f^3)$
Pe = Peclet number, $RePr$
Pr = Prandtl number, ν/α
Q_0'' = dimensionless constant heat flux applied at wall
Q_w^r = dimensionless wall radiative heat flux, $\pi q_w^r/(\tilde{n}^2\sigma T_f^4)$
q_0'' = constant heat flux applied at wall
q_w^r = wall radiative heat flux
R = radius of pipe
Re = Reynolds number, $u_m R/\nu$
r = radial coordinate
$\bar{r}$ = optical radial coordinate
S = source function
T = temperature
T_0 = inlet fluid temperature
T_f = reference temperature, $q_0''R/k$
u = velocity
u_m = maximum velocity
z = axial coordinate
$\bar{z}$ = optical axial coordinate
α = thermal diffusivity
η = dimensionless radial coordinate, r/R
θ = polar angle
κ = absorption coefficient
ν = kinematic viscosity
ξ = dimensionless axial coordinate, z/R
ρ = density
σ = Stefan-Boltzmann constant
τ_R = optical radius
ψ = dimensionless temperature, T/T_f
ψ_0 = dimensionless inlet fluid temperature
ψ_b = dimensionless bulk temperature
ψ_w = dimensionless wall temperature

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Introduction

THE problem of combined radiation and forced convection from a pipe wall to the fluid or vice versa is of importance in many industrial applications. They include heat exchangers in industry, the cooling processes in nuclear reactors, and solar energy collectors. Another example is the waste heat extraction from flue gases, which usually consist of particulates plus gaseous products. In the thermal analysis of such applications, interaction of radiation with convection must be considered in establishing the overall heat transfer. Accounting for the influence of radiation on the local heat transfer in ducts is a difficult problem and has been a subject of great interest for many years.

The interaction of thermal radiation and forced convection in the thermal entrance region of a circular pipe with sudden temperature jump has been studied by many investigators.[1-26] Only a few investigations have been concerned with combined radiation and forced convection in a circular tube with wall heat flux. Siegel and Perlmutter,[27,28] Chen,[29] Sikka and Iqbal,[30] and Ghoshdastidar and Bandyopadhyay[31] considered the nonparticipating medium flow through a circular pipe with uniform or nonuniform wall heat flux, and the heating section was semi-infinite. Greif and others[32-34] examined experimentally and theoretically fully developed turbulent flow of a nongray optically thin radiating gas subjected to a constant wall heat flux in a circular tube with black wall, and the heating section was also semi-infinite. Srivastava and Roux[35] examined the heat transport problem of a central receiver-type solar energy collector. Only absorption was considered for the working fluid.

The preceding literature review shows that only a few studies concerned with isothermal wall temperature boundary conditions have considered two-dimensional radiative heat transfer. Also, a study concerned with the wall heat flux boundary condition including two-dimensional radiation model is required. To simulate a more practical situation, the present study considers the interaction of thermal radiation and forced convection within a circular pipe flow with the wall heated by a uniform heat flux within a finite length and insulated otherwise. A two-dimensional radiative heat transfer model is considered in the present study, while the fluid is a radiatively absorbing and emitting medium and the tube wall is assumed gray and black.

Mathematical Analysis

In the present study, we deal with a steady hydrodynamically fully developed laminar flow of an incompressible, viscous, radiatively participating fluid in a circular pipe subjected to a constant heat flux (see Fig. 1). The fluid is an absorbing and emitting gray medium. The surface of the pipe is black.

We neglect the viscous dissipation in the fluid. We also neglect the axial conduction term. The energy equation in r-z coordinates is then given as

$$\rho C_p u \frac{\partial T}{\partial z} = k \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right) - \nabla \cdot q^r \tag{1}$$

where ρC_p is the heat capacity of the fluid. The vector q^r represents the net radiation heat flux present in the fluid. The velocity distribution u is assumed to be of parabolic form (i.e., laminar, hydrodynamically, fully developed).

For the present problem, the boundary conditions are

$$\frac{\partial T}{\partial r} = 0, \qquad \text{at } r = 0, \quad -\infty \leq z \leq \infty$$

$$k \left.\frac{\partial T}{\partial r}\right|_{r=R} + q_w^r = \begin{cases} q_0'', & 0 \leq z \leq l \\ 0, & \text{otherwise} \end{cases}$$

$$T(r,z) = T_0, \qquad \text{as } z \to -\infty$$

The energy equation and boundary conditions can be rewritten in terms of dimensionless quantities as

$$(1-\eta^2)\frac{\partial \psi}{\partial \xi} = \frac{1}{Pe}\left[\frac{1}{\eta}\frac{\partial}{\partial \eta}\left(\eta \frac{\partial \psi}{\partial \eta}\right) - \frac{\tau_R}{Nc}\nabla \cdot Q^r\right] \tag{2}$$

$$\frac{\partial \psi}{\partial \eta} = 0, \qquad \text{as } \eta = 0, \quad -\infty < \xi < \infty \tag{3a}$$

$$\left.\frac{\partial \psi}{\partial \eta}\right|_{\eta=1} + \frac{\tau_R}{4\pi Nc} Q_w^r = \begin{cases} 1, & 0 \leq \xi \leq L \\ 0, & \text{otherwise} \end{cases} \tag{3b}$$

$$\psi = \psi_0, \qquad \text{as } \xi \to -\infty \tag{3c}$$

Q_w^r is the dimensionless wall radiative heat flux and may be obtained in terms of the boundary intensity I_w and the radiation source function S as[36]

$$\begin{aligned} Q_w^r(\bar z) &= Q_{\text{inward}}^r(\bar z) - Q_{\text{outward}}^r(\bar z) = \pi I_{wb}(\bar z) \\ &- \left\{\int_{-\infty}^{\infty}\int_0^{\tau_R}\int_0^{2\pi} S(\bar r_3, \bar z_3)\exp(-d_3)\frac{\tau_R(\tau_R - \bar r_3 \cos\theta)}{d_3^3} \right. \\ &\times \mathrm{d}\theta\, \mathrm{d}\bar r_3\, \mathrm{d}\bar z_3 + \int_{-\infty}^{\infty}\int_0^{2\pi} I_w(\bar z_4)\exp(-d_4) \\ &\left. \times \frac{\tau_R^3(1-\cos\theta)^2}{d_4^4}\mathrm{d}\theta\, \mathrm{d}\bar z_4 \right\} \end{aligned} \tag{4}$$

where $\bar r$ and $\bar z$ are optical coordinates defined as the products of the medium absorption coefficient κ and geometry coordinates r and z, respectively. The source function S for nonscattering medium is

$$S(\bar r, \bar z) = I_b(\psi) = \psi^4 \tag{5}$$

and the boundary intensity I_w for the black wall is

$$I_w(\bar z) = I_{wb} = \psi^4(1,\xi) \tag{6}$$

The divergence of the net radiative heat flux vector in the energy equation is given in the $\bar r$-$\bar z$ coordinates by[36]

$$\begin{aligned} \nabla \cdot Q^r = 4\pi &\left[I_b(\psi) - \int_{-\infty}^{\infty}\int_0^{\tau_R}\int_0^{2\pi} S(\bar r_1, \bar z_1) \right. \\ &\times \exp(-d_1)\frac{\bar r_1\, \mathrm{d}\theta\, \mathrm{d}\bar r_1}{d_1^2}\mathrm{d}\bar z_1 \\ &\left. - \int_{-\infty}^{\infty}\int_0^{2\pi} I_w(\bar z_2)\exp(-d_2)\frac{\tau_R(\tau_R - \bar r\cos\theta)}{d_2^3}\mathrm{d}\theta\, \mathrm{d}\bar z_2 \right] \end{aligned} \tag{7}$$

where d in the preceding equations is expressed as

$$d_1 = d(\bar r, \bar z; \bar r_1, \bar z_1), \qquad d_2 = d(\bar r, \bar z; \tau_R, \bar z_2)$$

$$d_3 = d(\tau_R, \bar z; \bar r_3, \bar z_3), \qquad d_4 = d(\tau_R, \bar z; \tau_R, \bar z_4)$$

with $d(\bar r, \bar z; \bar r_i, \bar z_i) = [(\bar z - \bar z_i)^2 + \bar r^2 + \bar r_i^2 - 2\bar r \bar r_i \cos\theta]^{1/2}$ representing the distance between two points $(\bar r_i, \bar z_i, \theta)$ and $(\bar r, \bar z, 0)$.

The local Nusselt number is defined as

$$Nu(\xi) = \frac{2}{\psi_w - \psi_b} Q_0''(\xi)$$

where

$$Q_0''(\xi) = \begin{cases} 1, & \text{as } 0 \leq \xi \leq L \\ 0, & \text{otherwise} \end{cases}$$

and ψ_b is the dimensionless fluid bulk temperature given by

$$\psi_b(\xi) = 4\int_0^1 \psi(\eta, \xi)(1-\eta^2)\eta\, \mathrm{d}\eta \tag{8}$$

Solution Methodology

The governing equations for the present problem are a set of coupled equations, Eqs. (2–7). The Crank-Nicolson finite difference method is used to solve the axial differential part, and the central difference is used in the radial differential part. A second-order accuracy can be expected in computing the corresponding finite difference equations of Eqs. (2) and (3). To construct the approximations of divergence of radiative heat flux, $\nabla \cdot q^r(\bar r, \bar z)$, and net wall radiative heat flux, $Q_w^r(\bar z)$, the finite element nodal approximation method is utilized.[37] In the method, the whole geometric domain, $0 \leq r \leq R$ and $-\infty \leq z \leq \infty$, is subdivided into rectangular elements. The linear approximations over each element are considered for the radiation source function and boundary intensity. The expression for the net wall radiative heat flux and the divergence of net radiative heat flux vector, Eqs. (4) and (7), can then be described in terms of the integrals over these finite elements.

In numerical computation, the expression for wall radiative heat flux is rewritten as in the following by properly choosing values of distance L_1 and L_2 that are large enough so that the source function and boundary intensity at nodes

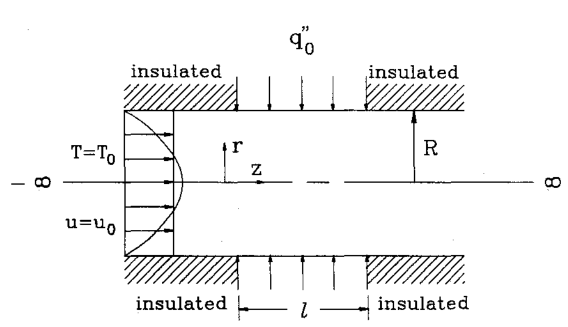

Fig. 1 Coordinate system and physical model.

outside the range $-L_1<z<L_2$ can be assumed independent of $\bar{z}$ coordinate:

$$
\begin{aligned}
Q_w^r(\bar{z}) &= Q_{\text{inward}}^r(\bar{z}) - Q_{\text{outward}}^r(\bar{z}) \\
&= \pi I_{wb}(\bar{z}) - \left\{ \sum_{i=1}^{n} \int_{\bar{r}_{1,i-1}}^{\bar{r}_{1,i}} S^1(\bar{r}_3) \int_{-\infty}^{-L_1} \int_0^{2\pi} \exp(-d_3) \right. \\
&\times \frac{\tau_R(\tau_R-\bar{r}_3\cos\theta)}{d_3^3}\, d\theta\, d\bar{z}_3\, d\bar{r}_3 - \sum_{j=1}^{N}\sum_{i=1}^{n} \\
&\times \int_{\bar{z}_{1,j-1}}^{\bar{z}_{1,j}} \int_{\bar{r}_{1,i-1}}^{\bar{r}_{1,i}} \int_0^{2\pi} S(\bar{r}_3,\bar{z}_3)\exp(-d_3)\frac{\tau_R(\tau_R-\bar{r}_3\cos\theta)}{d_3^3} \\
&\times d\theta\, d\bar{r}_3\, d\bar{z}_3 + \sum_{i=1}^{n} \int_{\bar{r}_{1,i-1}}^{\bar{r}_{1,i}} S^2(\bar{r}_3) \int_{L_2}^{\infty} \int_0^{2\pi} \exp(-d_3) \\
&\times \frac{\tau_R(\tau_R-\bar{r}_3\cos\theta)}{d_3^3}\, d\theta\, d\bar{z}_3\, d\bar{r}_3 + I_{w1}\int_{-\infty}^{-L_1}\int_0^{2\pi}\exp(-d_4) \\
&\times \frac{\tau_R^3(1-\cos\theta)^2}{d_4^4}\, d\theta\, d\bar{z}_4 + \sum_{j=1}^{N}\int_{\bar{z}_{1,j-1}}^{\bar{z}_{1,j}}\int_0^{2\pi} I_w(\bar{z}_4)\exp(-d_4) \\
&\times \frac{\tau_R^3(1-\cos\theta)^2}{d_4^4}\, d\theta\, d\bar{z}_4 + I_{w2}\int_{L_2}^{\infty}\int_0^{2\pi}\exp(-d_4) \\
&\left. \times \frac{\tau_R^3(1-\cos\theta)^2}{d_4^4}\, d\theta\, d\bar{z}_4 \right\}
\end{aligned}
$$

The subscript $1,i$ in the preceding expression denotes the coordinate of ith node in that direction. The linear approximations of $S(\bar{r},\bar{z})$ and $I_w(\bar{z})$ are, respectively, given as

$$
\begin{aligned}
S(\bar{r},\bar{z}) &= S_{(i-1,j-1)}(1-xx)(1-yy) + S_{(i-1,j)}(1-xx)yy \\
&+ S_{(i,j-1)}xx(1-yy) + S_{(i,j)}xx\cdot yy
\end{aligned}
$$

and

$$
I_w(\bar{z}) = I_{w,j-1}(1-yy) + I_{w,j}(yy)
$$

where $xx=(\bar{r}-\bar{r}_{1,i-1})/(\bar{r}_{1,i}-\bar{r}_{1,i-1})$ and $yy=(\bar{z}-\bar{z}_{1,j-1})/(\bar{z}_{1,j}-\bar{z}_{1,j-1})$. $S_{(i,j)}=S(\bar{r}_{1,i},\bar{z}_{1,j})$ and $I_{w,j}=I_w(\bar{z}_{1,j})$ are the nodal parameters. Also, $\bar{r}_{1,0}=0$, $\bar{r}_{1,n}=\tau_R$, $\bar{z}_{1,0}=-L_1$, and $\bar{z}_{1,N}=L_2$; $S^1(\bar{r})=S(\bar{r},-L_1)$, $S^2(\bar{r})=S(\bar{r},L_2)$, $I_{w1}=I_w(-L_1)$, and $I_{w2}=I_w(L_2)$. The approximations of S^1 and S^2 are given as

$$
S^1(\bar{r}) = S_{(i-1,0)}(1-xx) + S_{(i,0)}xx
$$

$$
S^2(\bar{r}) = S_{(i-1,N)}(1-xx) + S_{(i,N)}xx
$$

The same treatment is applied to the expression for divergence of net radiative heat flux vector. The eight-point Gaussian quadrature is used to calculate the encountered integration. To solve a problem, a guessed temperature distribution is substituted into Eqs. (5) and (6) to obtain the source function and boundary intensity at nodes. And then $\nabla\cdot Q^r(\bar{r},\bar{z})$ and $Q_w^r(\bar{z})$ can be solved with the obtained source function and boundary intensity. Subsequently, the governing energy equation with the boundary conditions can be solved with the divergence of radiative heat flux, and net wall radiative heat flux and a new temperature distribution are then obtained. The procedure is repeated until the convergence criterion is satisfied. The under-relaxation factor is applied in the procedure of iteration to guarantee a convergent solution. Since the grid points taken for the radiation part are fewer than the convection part, a linear interpolation is required between the grid points of the radiation part to obtain the divergence of radiative heat flux at each grid point of the convection part.

Table 1 Numerical experiment on the number of radial node points and axial step size at $Pe=100$, $Nc=0.1$, $\psi_0=0.5$, and $L=1.0$

z		0.0	0.5	1.0
		$\tau_R=0.5$		
ψ_b	Case 1	0.50287	0.52082	0.53712
	Case 2	0.50290	0.52091	0.53722
ψ_w	Case 1	0.56468	0.71346	0.76766
	Case 2	0.56507	0.71404	0.76672
		$\tau_R=1.0$		
ψ_b	Case 1	0.50309	0.52232	0.53921
	Case 2	0.50318	0.52210	0.53864
ψ_w	Case 1	0.56096	0.68900	0.73633
	Case 2	0.56103	0.68870	0.73450

$(n,\Delta\bar{z})$: Case 1, (5,0.2); Case 2, (6,0.15).

Results and Discussion

The energy equation is solved by the Crank-Nicolson finite difference method with the axial step size 0.1–0.01. The step size is larger far from the heating section and gets smaller nearer the heating section. The step size is 0.01 throughout the heating section. The radial direction is divided uniformly into 51 grids and solved by a central-difference method. The results of bulk-temperature calculations by the aforementioned grid system have been compared to the analytic solution for a nonradiation case and show excellent agreement. There are five nodes in the radial direction for computing divergence of radiative heat flux, $\nabla\cdot Q^r$, and net wall radiative heat flux, Q_w^r for $\tau_R=0.5$. The axial step size is 0.2 optical distance in computing $\nabla\cdot Q^r$ and Q_w^r. A smaller grid size (six nodes in the radial direction and an axial step size of 0.15) makes no significant difference in the results, as shown in Table 1. Therefore, the grid system $(m,\Delta z)=(50,0.1\text{–}0.01)$ is applied for the convection part and $(n,\Delta\bar{z})=(5,0.2)$ for the radiation part in this study. The iterations are performed until the relative error of net wall radiative heat flux is less than 10^{-4} at all nodes between two successive iterations. Here, we choose Q_w^r as the convergent criterion because the convergence of Q_w^r guarantees the convergence of temperature.

Since the Nusselt number is just another representation of $\psi_w-\psi_b$, we only present the temperature profiles and wall radiative heat flux in the following. Figure 2 illustrates the effect of the conduction to radiation parameter (Nc) on the bulk temperature (ψ_b), wall temperature (ψ_w), and the wall radiative heat flux (Q_w^r) for $Pe=100$, $L=1.0$, $\psi_0=0.5$, and $\tau_R=0.5$. The Nc generally expresses the significance of radiation effect relative to conduction in medium. Since Nc is defined based on the reference temperature, the change in wall heat-flux levels would also affect the reference temperature which in turn would result in different Nc values. A small Nc represents a strong effect of radiation or a higher wall heat-flux rate, and thus more energy is transferred through radiation from the heating section, as shown in Fig. 2b. Since radiation is a more straightforward heat transport compared with other heat transfer modes, it is also noticed in Fig. 2a that there exists a strong preheat and postheat due to the strong effect of radiation or a higher wall heat flux.

The effects of optical radius on the bulk temperature, wall temperature, and wall radiative heat flux are shown in Fig. 3 for fluid at $Pe=100$, $Nc=0.1$, $L=1.0$, and $\psi_0=0.5$. With increasing τ_R, the medium becomes optically thicker and the radiation gets more difficult to transmit. The word "thicker" can be interpreted as higher density and thus the medium becomes more strongly absorbing. A medium with large optical radius absorbs more heat from the heating wall through radiation, as shown in Fig. 3b. Figure 3a also shows that the strong preheat and postheat appear at a large optical radius. This is because the fluid in both upstream and downstream regions absorbs more heat directly from the heating wall through radiation. The case $\tau_R=0$ implies that the medium is transparent to radiation, i.e., the medium cannot absorb and

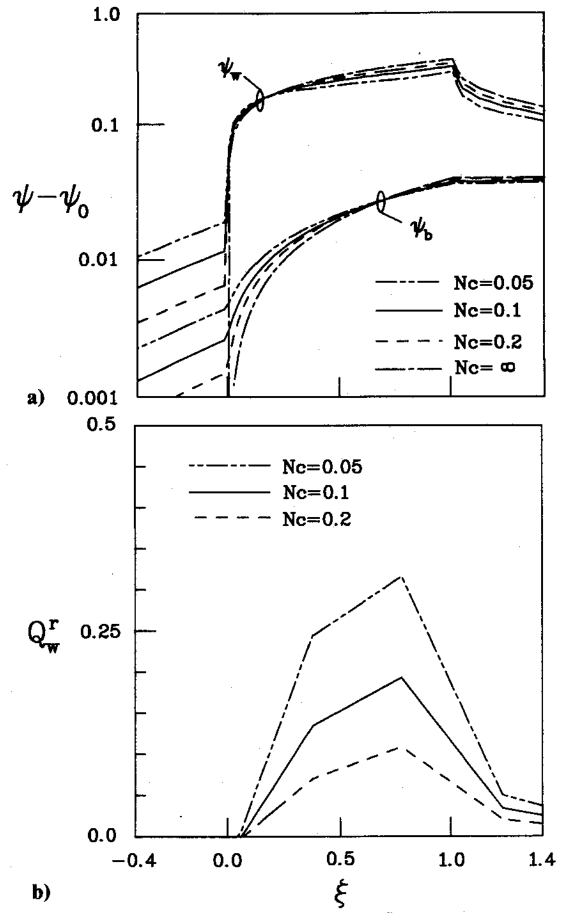

Fig. 2 Effect of Nc on a) ψ_b and ψ_w and b) Q_w^r for $Pe = 100$, $\tau_R = 0.5$, $L = 1.0$, and $\psi_0 = 0.5$.

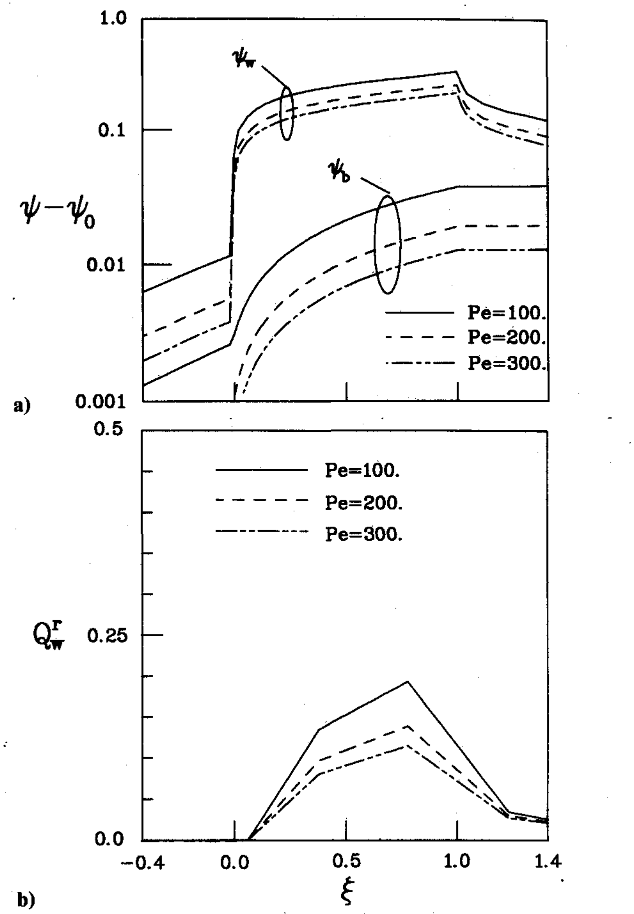

Fig. 4 Effect of Pe on a) ψ_b and ψ_w and b) Q_w^r for $Nc = 0.1$, $\tau_R = 0.5$, $L = 1.0$, and $\psi_0 = 0.5$.

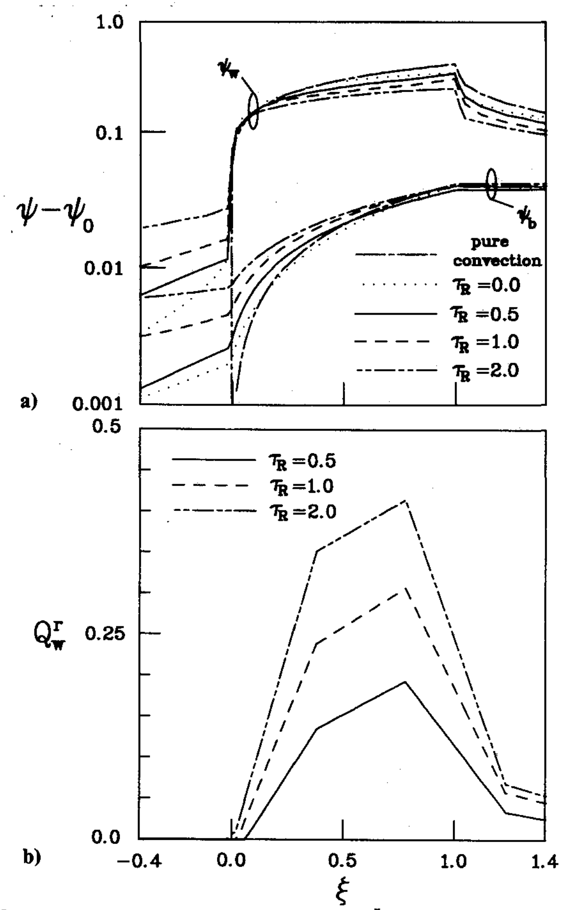

Fig. 3 Effect of τ_R on a) ψ_b and ψ_w and b) Q_w^r for $Pe = 100$, $Nc = 0.1$, $L = 1.0$, and $\psi_0 = 0.5$.

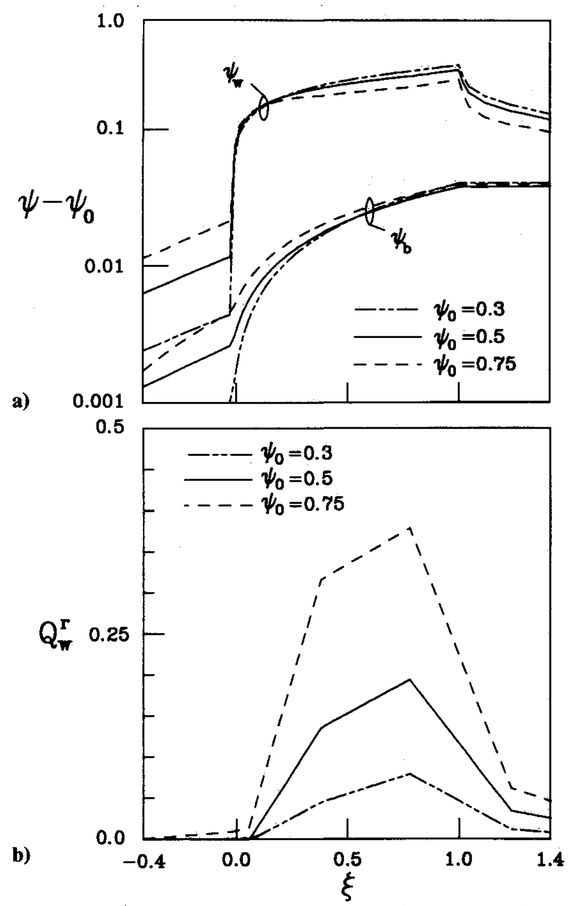

Fig. 5 Effect of ψ_0 on a) ψ_b and ψ_w and b) Q_w^r for $Pe = 100$, $Nc = 0.1$, $\tau_R = 0.5$, and $L = 1.0$.

scatter radiation. For such a case, Nc is also zero except when $R=0$ and the dimensionless boundary condition, Eq. (3b), cannot apply. The results for $\tau_R=0$ depend on $(4R\bar{n}^2\sigma T_f^3)/k$ rather than τ_R/Nc in Eqs. (2) and (3). We have obtained results for $\tau_R=0$ at an arbitrary value of $(4R\bar{n}^2\sigma T_f^3)/k$. Comparison of the results with that for pure convection shows that wall-to-wall radiation at $\tau_R=0$ smoothens the temperature profiles, as shown in Fig. 3a.

The effects of convective Peclet number on bulk temperature, wall temperature, and radiative wall heat flux are shown in Fig. 4 for the case with $Nc=0.1$, $L=1.0$, $\psi_0=0.5$, and $\tau_R=0.5$. A higher Peclet number represents a higher mass flow rate or heat capacity of the fluid medium. A fluid with higher Peclet number requires more energy to increase its own bulk temperature because of high heat capacity or high mass flow rate. The bulk temperature and wall temperature thus decrease for a given wall heat flux as Peclet number increases, as shown in Fig. 4a. Besides, a weaker preheat and postheat of the medium as the Peclet number is getting larger are also shown in Fig. 4a. The radiative wall heat flux decreases as Peclet number increases, as shown in Fig. 4b. This is because a higher Peclet number causes a high conductive heat flux at the wall and thus reduces radiative wall heat flux for a constant total wall heat flux.

Radiative heat transfer, unlike convective or conductive heat transfer, is highly nonlinear (fourth power of temperature) in nature and is emitted by virtue of nonzero absolute temperature of the media. This accounts for the difficulties in the numerical analysis. In addition, the absolute values of the temperatures also affect the physical phenomena. The influences of inlet fluid temperature ψ_0 upon bulk temperature, wall temperature, and wall radiative heat flux for $Nc=0.1$, $Pe=100$, $L=1.0$, and $\tau_R=0.5$ are shown in Fig. 5. It is noticed in Fig. 5a that the temperature increases at various inlet fluid temperatures have only slight differences at a given wall heat flux. However, due to the nonlinear characteristics of radiation transfer, the fraction of energy which is transferred through radiation from the heating section is profoundly increased as inlet temperature increases, as shown in Fig. 5b. This indicates that at higher dimensionless inlet temperatures, radiation becomes more dominating. As a consequence, the temperature profiles are more smoothened, as shown in Fig. 5a. That is, a good preheat and postheat exist for a fluid with higher dimensionless inlet temperature. It would be worthy to point out here that the change in wall heat flux may result in different dimensionless inlet temperature since the normalization is with respect to $(q_0''R/k)$.

Conclusions

The interaction between radiation and laminar forced convection heat transfer in a radiatively participating fluid flow in a circular pipe with a finite length heating section and the effects of varous radiation and convection parameters on fluid bulk temperature, wall temperature, and wall radiative heat flux have been analyzed in this study. The results obtained can be briefly summarized as follows:

1) A small conduction-to-radiation parameter and/or a large optical radius cause a high wall radiative heat flux and low wall temperature. Besides, a good preheat and postheat appears at a small conduction-to-radiation parameter and large optical radius.

2) The decrease in Peclet number increases wall radiative heat flux and causes a good preheat and postheat.

3) At higher inlet fluid temperature radiation becomes more dominating. A good preheat and postheat thus exist.

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